



**Welcome to the 19<sup>th</sup> International Workshop for Young Mathematicians organized by the Zaremba Society of Mathematicians – Students of the Jagiellonian University and Faculty of Mathematics and Computer Science of the Jagiellonian University.**

The International Workshop for Young Mathematicians gives young researchers, undergraduate and PhD students the opportunity to give a talk, in which they can present the results of their research. This year the Workshop is devoted to Algebraic Geometry. The talks will be given by students and renowned specialists in the field. The Workshop's program also includes poster presentations and a wide variety of social activities, which we hope will help bring participants together.

During the 19<sup>th</sup> Workshop we will also listen to lectures given by prominent mathematicians. Our invitation was accepted by:

**Gilberto Bini (Università degli Studi di Milano)**  
**Alessandra Sarti (Université de Poitiers)**  
**Yukihide Takayama (Ritsumeikan University)**

The scientific committee of the Workshop consists of researchers of the Jagiellonian University:

**Sławomir Cynk**  
**Marcin Dumnicki**  
**Michał Kapustka**

We hope that the 19<sup>th</sup> Workshop will once again demonstrate that not only is mathematics beautiful, but it is also useful in many domains of human life. We believe that the atmosphere of Kraków will help you establish new relations.



## ORGANIZING COMMITTEE

Magdalena Bera  
Renata Furgał  
Anita Gilarska  
Julia Kaznowska  
Adam Koroński  
Natalia Kupiec  
Antoni Machowski  
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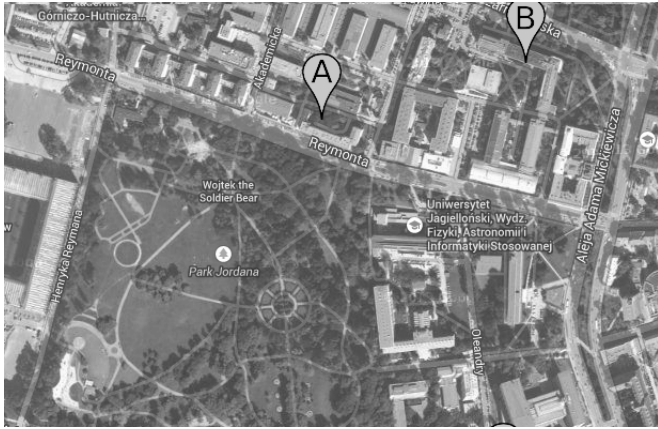
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of Mathematics and Computer  
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Włodzimierz Zwonek




-  Accommodation  
*Nawojka Student Hotel*
-  Canteen (Breakfasts)  
*Building C3, AGH Campus*




 Barbecue on Monday, Integration Evening on Friday  
*Ryba Babel Club (Piast Café)*



Lecture Room, Conference Office

 *Faculty of Mathematics and Computer Science of the Jagiellonian University*  
Canteen (Lunches)

 *Faculty of Physics, Astronomy and Applied Computer Science of the Jagiellonian University*



## INFORMATION FOR PARTICIPANTS

The **Conference Office** is located at the Faculty of Mathematics and Computer Science of the Jagiellonian University (address: Łojasiewicza 6). The office is open:

from 10:00 a.m. to 11:00 a.m. and during coffee breaks  
(September 19<sup>th</sup>),  
from 9:45 a.m. to 10:00 a.m. and during coffee breaks  
(September 20<sup>th</sup>–23<sup>rd</sup>),  
from 9:45 a.m. to 12:00 p.m. (September 24<sup>th</sup>).

**Registration for participants** will start on Monday, September 19<sup>th</sup>, during the office opening hours.

**Accommodation** in the Nawojka Student Hotel (address: Reymonta 11) starts at 2 p.m. on Sunday, September 18<sup>th</sup> and ends at 12 p.m. on Saturday, September 24<sup>th</sup>.

**Breakfasts** will be served at the canteen in building C3 at the AGH Campus and **lunches** will be served at the Faculty of Physics, Astronomy and Applied Computer Science of the Jagiellonian University (address: Łojasiewicza 11). The first meal will be a breakfast on Sunday, September 18<sup>th</sup> and the last one – a lunch on Saturday, September 24<sup>th</sup>.

**Lectures** will be given at the Faculty of Mathematics and Computer Science of the Jagiellonian University. We kindly ask Lecturers to come to the Lecture Room at least ten minutes before their speech.

**To travel between the Nawojka Student Hotel and the Faculty of Mathematics and Computer Science of the Jagiellonian University** we recommend using buses no. 194. To get to the lecture room you should go to the AGH bus stop (Aleja Adama Mickiewicza) and get off the bus at Norymberska stop. Then go towards Kaufland and along the building of Faculty of Management and Social Communication of the Jagiellonian University. The Faculty of Mathematics and Computer Science is yellow building

behind it. To get back to the hotel get in the bus at the Norymberska stop and get off at Czarnowiejska stop. Move back to Aleja Adama Mickiewicza, pass AGH main building and go to Reymonta. You can shorten the trip by going through the AGH campus.

Each participant of 19<sup>th</sup> International Workshop for Young Mathematicians will have an opportunity to take part in the following **social and cultural events**:

**Kraków City Evening Tour** (Sunday, 6:30 p.m.–11:00 p.m.). The tour begins in Main Market Square by the Statue of Adam Mickiewicz.

**Integration Barbecue Evening** (Monday, 7:00 p.m.–12:00 a.m.). The party will take place at the Ryba Babel Club called also Piast Café (address: Piastowska 47).

**Bowling and Billiard Evening** (Tuesday, 8:00 p.m.–10:00 p.m.). We will spend this evening in the Plaza Centre (address: Aleja Pokoju 44). We will gather before the event and travel to the place together. You will be able to find out the details at the information desk.

**Karaoke Evening** (Wednesday, 8:00 p.m.–12:00 a.m.). We will go to Coyote Bar (address: Św. Tomasza 1) and everyone will be able to try singing in public. There is no need to be embarrassed!

**Laser paintball** (Thursday, 8:00 p.m.–9:00 p.m.). This event takes place at Prokocimska 8. The information will be given during the lectures.

**Integration Evening** (Friday, 7:00 p.m.–11:30 p.m.). The party will take place in the Ryba Babel Club (address: Piastowska 47).

Participation in all events is **free of charge** for Workshop participants. We strongly encourage you to take part in them!





## WORKSHOP SCHEDULE

All lectures will take place at the Faculty of Mathematics and Computer Science of the Jagiellonian University (address: Łojasiewicza 6).

### SUNDAY – September, 18<sup>th</sup>;

15:00 – 16:00	REGISTRATION
18:30 – 21:00	KRAKÓW SIGHTSEEING TOUR

### MONDAY – September, 19<sup>th</sup>;

8:00 – 9:00	BREAKFAST
10:00 – 11:00	REGISTRATION
11:30 – 12:30	Gilberto Bini, Lecture
13:00 – 14:00	Gilberto Bini, Lecture
14:00 – 15:00	LUNCH
15:00 – 16:00	Gilberto Bini, Classes
19:00 – 23:00	GRILL PARTY

### TUESDAY – September, 20<sup>th</sup>;

8:00 – 9:00	BREAKFAST
10:00 – 11:00	Alessandra Sarti, Lecture
11:30 – 12:30	Alessandra Sarti, Lecture
13:00 – 14:00	Marcin Sroka, <i>Where Differential Geometry meets Algebraic Geometry</i>
14:00 – 15:00	LUNCH
15:00 – 16:00	Alessandra Sarti, Classes
20:00 – 22:00	BOWLING / BILLART

**WEDNESDAY – September, 21<sup>st</sup>;**

8:00 – 9:00	BREAKFAST
10:00 – 11:00	Alessandra Sarti, Lecture
11:30 – 12:30	Arkadiusz Bochniak, <i>Hopf algebras and theory of motives in physics</i> (part 1)
11:30 – 12:30	Arkadiusz Bochniak, <i>Hopf algebras and theory of motives in physics</i> (part 2)
14:00 – 15:00	LUNCH
15:00 – 16:00	Anna Parlak, <i>Groups of Automorphisms of Hyperelliptic Riemann Surfaces</i>
20:00 – 23:00	KARAOKE

**THURSDAY – September, 22<sup>nd</sup>;**

8:00 – 9:00	BREAKFAST
10:00 – 11:00	Yukihide Takayama, Lecture
11:30 – 12:30	Yukihide Takayama, Lecture
13:00 – 14:00	POSTER SESSION
14:00 – 15:00	LUNCH
15:00 – 16:00	Yukihide Takayama, Lecture
20:00 – 21:00	LASER TAG

**FRIDAY – September, 23<sup>rd</sup>;**

8:00 – 9:00	BREAKFAST
10:00 – 11:00	Błażej Żmija, <i>Basic facts about elliptic curves with complex multiplication</i>
11:30 – 12:30	Dominik Burek, <i>Invariants of Calabi–Yau 3 folds of Borcea–Voisin type</i>
13:00 – 14:00	Jędrzej Garnek, <i>Elliptic curves over rings</i>
14:00 – 15:00	LUNCH
15:00 – 16:00	CLOSING CEREMONY
19:00 – 23:00	FAREWELL PARTY

**SATURDAY – September, 24<sup>th</sup>;**

8:00 – 9:00	BREAKFAST
14:00 – 15:00	LUNCH



## ABSTRACTS

### Hopf algebras and theory of motives in physics

ARKADIUSZ BOCHNIAK

In quantum field theory there is a fundamental problem related to the calculation of integrals obtained from Feynman rules associated with given perturbative Quantum Field Theory (pQFT). These integrals are often divergent, but it is well-known that there is a physical information contained in such objects. To obtain these hidden information we need process of renormalization. This problem was partially solved by introduction so-called BPHZ formalism, but it was a kind of mystery why it works. Kreimer and Connes shown that it can be described using Hopf algebras and Rota-Baxter systems related to rooted planar trees. Moreover, Feynman amplitudes can be also described using formalism which follows from the theory of motives. Concept of motives was introduced by A.Grothendieck. We want smooth projective varieties over a field to be additive category. Using three-step construction (linearization, pseudoabelianization and inversion) we can construct such a rigid category. Using this formalism problems in pQFT can be presented (and partially solved) in the language of algebraic geometry using Galois groups formalism, Weil cohomology, Tannakian categories and Hodge structures.

## Invariants of Calabi–Yau 3 folds of Borcea–Voisin type.

DOMINIK BUREK

Calabi – Yau 3 fold obtained as a crepant resolution of  $S \times E/(\alpha_S \times \alpha_E)$ , where  $\alpha_S \in \text{Aut}S$  and  $\alpha_E \in \text{Aut}E$  are purely nonsymplectic automorphisms is called Borcea – Voisin type. In *Calabi-Yau 3-folds of Borcea–Voisin type and elliptic fibrations* A. Cattaneo and A. Garbagnati computed Hodge numbers of such 3-folds for automorphisms of order 2,3,4 and 6. We want to get the same results using orbifold formulas introduced by Chen and Ruan in *A new cohomology theory of orbifold*.

## Elliptic curves over rings

JĘDRZEJ GARNEK

The starting point for this lecture will be a problem concerning the  $p$ -torsion of elliptic curves over  $\mathbb{Q}_p$ , the field of  $p$ -adic numbers. It turns out that some information about the torsion depends only on reduction modulo some high power of the prime  $p$ . To formalize this statement, we need the notion of an elliptic curve over an arbitrary ring. Elliptic curves over rings can be viewed using Weierstrass equations (at least locally), but the scheme-theoretic approach is usually more fruitful. In the lecture we will give some properties of such curves and try to classify them.

## Groups of Automorphisms of Hyperelliptic Riemann Surfaces

ANNA PARLAK

*Riemann surfaces* are compactified sets of solutions to complex polynomial equations in two variables. To each automorphism of a Riemann surface  $X$ , that is a bijective function  $\varphi : X \rightarrow X$  which preserves the complex structure, corresponds an algebraic automorphism of the equation describing  $X$  — i.e. nontrivial

change of variables which does not change the equation. The typical examples of Riemann surfaces admitting a nontrivial automorphism are *hyperelliptic Riemann surfaces*. They can be represented by equations of the form  $y^2 = F(x)$ . One can make a substitution  $(x', y') = (x, -y)$  and the equation does not change. This means that there exists a nontrivial automorphism, the so-called *hyperelliptic involution*  $\rho$ , of the Riemann surface corresponding to the solution set of this equation. We can visualise it as a rotation by  $180^\circ$  about  $y$ -axis.

Topologically we can consider a group of isotopy classes of self-homeomorphism of a surface, which we call the *mapping class group*. The centraliser of  $[\rho]$  in this group is called the *hyperelliptic mapping class group*. Even though these objects ignore the complex structure, they are useful in the study of automorphisms of Riemann surfaces because of a positive solution to the Nielsen Realisation Problem. I will discuss these interconnections between algebraic geometry and topology and also give some explicit examples of equations for hyperelliptic curves with a given automorphism group.

## Where Differential Geometry meets Algebraic Geometry

MARCIN SROKA

Lecture will focus on situations where smooth algebraic varieties serve as examples of smooth manifolds with certain additional structure. We will explore some concrete examples of such situations (ex. Calabi-Yau or Hyperkahler manifolds). Some general discussion will be done as well.

## Basic facts about elliptic curves with complex multiplication

BŁAŻEJ ŻMIJA

For an elliptic curve  $E/\mathbb{Q}$  given by  $y^2 = f(x)$  it is important to know the number of solutions of the equation  $y^2 = f(x)$  modulo any prime number  $p$ . For this purpose we introduce the following number:

$$a_p := p - \#\{ (x, y) \in \mathbb{F}_p^2 \mid y^2 = f(x) \} = - \sum_{x=0}^{p-1} \left( \frac{f(x)}{p} \right),$$

where  $\left( \frac{\cdot}{p} \right)$  denotes the Legendre symbol. In the case of elliptic curve with complex multiplication, there is an exact formula for  $a_p$ .

In this talk we present the motivation behind theory of elliptic curves with complex multiplication and we prove the formulas for  $a_p$  in the simplest two cases (with automorphisms of order 4 or 6). Our paper is based on the paper 'Aspects of complex multiplication' by Don Zagier.



## POSTERS

### **Calabi-Yau Manifolds**

MARCIN SROKA

The poster will present chosen facts about a famous class of manifolds with specific geometric structure - namely Calabi-Yau manifolds. Some discussion about examples of them will be presented as well.

### **Around Hilbert's 17th Problem**

BEATA MOCKIEWICZ, KAROLINA MROCYŃSKA

Hilbert's 17th problem is one of the 23 Hilbert problems set out in a celebrated list compiled in 1900 by David Hilbert. It concerns the expression non-negative definite rational functions as finite sums of squares of rational functions. The first solution was due to Emil Artin in 1927, who used the theory of real closed fields. The poster will present the history of solving Hilbert's 17th problem and some notes on real closed fields.



# **Algebraic Geometry in Cryptography**

WIESŁAW MALESZEWSKI

The paper presents the application of algebraic geometry in cryptography. In the first part we begin with basic issues such as elliptic curves, next we learn the various cryptographic systems based on elliptic curves, and then we come to the other families of curves used in cryptography, which are presented with cryptographic algorithms that are based on those curves. At the end we show some examples of applications of these methods of information protection that are used in the modern world.



## HISTORY OF THE WORKSHOP

The International Workshop for Young Mathematicians is a conference organized by Zaremba Society of Mathematicians – Students of the Jagiellonian University. It is intended especially for graduate and Ph.D. students of mathematics and other related disciplines. Scientific workers are also welcome.

The beginnings of the Workshop go back to 1997, when a group of enthusiasts under the leadership of Anna Stasica, then the president of the Society, organized the first meeting of this type. There were only 15 students, mainly from Kraków, who took part in it. During a week, in five-hour blocks of lectures and classes, the participants were penetrating the mysteries of algebraic and analytic geometry. They were supervised by the workers of Polish Academy of Sciences and the Department of Algebraic and Analytic Geometry of the Jagiellonian University.

The beginnings were modest, but the news about a conference for young mathematicians spread widely. 53 people took part in the 2<sup>nd</sup> Workshop. It dealt with differential equations and dynamical systems. Then the first award for the best lecture was presented. Also, the first note in the press occurred: *Gazeta Wyborcza* (a popular Polish newspaper) appreciated the effort of Jagiellonian students.

The 3<sup>rd</sup> National Workshop for Young Mathematicians, Singularity Theory, fully deserved its name. It brought together 73 students from 19 Polish colleges. Also, the first guests from abroad came: students from two universities in L'viv, and lecturers from Belgium and France. An integration meeting became a tradition, and a bonfire as well, although people were more and more cramped around it.

The path had been smoothed, but the organization of the Workshop still required a lot of effort. There were innumerable applications, agreements and reservations that needed to be prepared already in November... and the number of participants was growing year by year! 86 people participated

in the 4<sup>th</sup> edition, which was devoted to complex analysis. Many of them were already our regular guests.

The 5<sup>th</sup> Workshop, Approximation Theory, turned out to be a success. There were 104 participants from 22 colleges. It seemed impossible to integrate such a mass of people. But it succeeded! The majority of our guests promised enthusiastically to come the following year... and they came! The record for the number of participants was beaten again: 142 students from 42 universities, including 22 guests from abroad, attended the 6<sup>th</sup> Workshop for Young Mathematicians, Operator Theory.

Because of the growing number of the latter, the next edition of the Workshop, devoted to applied mathematics, became International in its name too. English became a dominating language of the conference. Also, the first poster session was organized then.

In 2007 we found out more about Combinatorics. 204 participants arrived to our Workshop. In order to celebrate the 10th birthday of our conference, we organized a first open problem session in the history of the Workshop as well as HEX tournament. The next Workshop (2008) was devoted to Number Theory. We organized the second open problem session as well as a board games evening. During the 2009 Workshop (Probability Theory and Statistics) the menu of entertainments grew even broader: we spent one evening playing paintball match. During the last six Workshops (2010–Logic and Foundations of Mathematics, 2011–Algebra, 2012–Functional Analysis, 2013–Ergodic Theory and Dynamical Systems, 2014–Discrete Mathematics, 2015–Algebraic and Differential Topology) we were having a great time playing billiard and bowling. In 2012, participants of the Workshop enjoyed a Yoga Class for the first time. Since 2014 the Workshop takes place in new building at Łojasiewicz Street.



## A FEW WORDS ABOUT OUR SOCIETY

The Zaremba Society of Mathematicians-Students of Jagiellonian University is one of the oldest scientific organizations at the Jagiellonian University. Not only does it assemble students of mathematics, but it also has many supporters outside the Institute of Mathematics.

The Society has existed already for over 120 years. It was established in 1893 (under a different name). From the beginning of its existence, it organized numerous talks, and already in 1902, it started publishing. At first it used to publish only scripts, and books later on. A period of extraordinary growth of the Society came between the two World Wars, when the number of its members reached 200. Many famous Polish mathematicians were involved in its activity, including Stanisław Zaremba, Witold Wilkosz, Stanisław Gołąb, Andrzej Turowicz and Kazimierz Kordylewski. The Society played a considerable role during Hitler's occupation: its library was providing books for secret teaching and research. After the War, the Society had been working actively until 1950. Then, its activity was banned, and its office was closed and sealed. But the formal closure of the Society has never happened. It kept working unofficially during the next nine years. Then, it was reactivated in March 1959. It was decided that besides scientific initiatives, the Society should organize cultural, social and touristic events. In 1973, it was named for Professor Stanisław Zaremba, a great mathematician of Kraków.

The statutory job of the Society is to organize students' scientific life. We try hard to achieve this goal by organizing various initiatives. Apart from the annual International Workshop for Young Mathematicians, we organize Numbers – Computers – Life – bioinformatics conference, which became quite popular among students and researchers in related fields. The other events and activities organized regularly by our society are: summer and winter holiday camps with mathematical and leisure activities, weekend trips for taking a rest and playing with maths, student sessions where we talk about what is interesting for us, as well as scientific meetings with a cup of tea or

coffee, going back to the tradition of the Scottish Café in L'viv. We also participate actively in various other nationwide scientific initiatives.

Moreover, we organize football and volleyball *Professors vs. Students* matches, bridge and chess tournaments, St Andrew's Eve and carnival parties. The stormy and fascinating history of the Society is well-documented in numerous chronicles. They are accessible in the Society office which is a place where one can drink some tea or have a chat, while students of early years can obtain some help.

We have our own large library containing old and valuable volumes as well as textbooks that are most useful for students. We gather our professors' interesting sayings, and publish some comic booklets. Every year we publish proceedings of the previous Workshop.

