MINI-COURSES

An introduction to the classification of smooth manifolds

Matthias Kreck, Peter Teichner

The aim of the lectures and problem sessions is to give an introduction to some aspects of the classification of manifolds. We plan to give surveys of the following topics:

(1) Basic results about manifolds, like the Whitney embedding theorem.

(2) Surfaces, projective spaces, lens spaces, plumbings, Heegard decompositions, knot complements, mapping tori, thickenings of CW-complexes.

(3) Algebraic invariants: fundamental group, Euler characteristic, intersection form, signature.

(4) Morse functions and corresponding handle body decompositions.

(5) Classification of closed manifolds of dimension < 4.

(6) Manifolds up to cobordism and translation to homotopy theory.

(7) The s-cobordism theorem and exotic 7-spheres.

(8) Stable classification of manifolds and the relation to cobordism groups.

(9) Classification of 4-manifolds with special fundamental groups and some exotic 4-manifolds.

The Problem Session will be run with the help of **Markus Land** and **Daniel Kasprowski**.

Compact Lie Group Actions on Smooth Manifolds

Krzysztof Pawałowski

The first lecture will last 90 minutes. The following topics will be discussed during the lecture.

(1) Haar integral and orthogonal representations of compact Lie groups.

(2) Cells and handles with group actions, and vector bundles over orbits.

(3) Vector bundles over contractible CW complexes in the equivariant setting.

(4) Smooth manifolds as CW complexes with vector bundles in the equivariant setting.

(5) The Mostow–Palais embedding theorem.

The problem session will last for 90 minutes.The participants will solve problems related to Lecture 1, as well as those needed for Lecture 2. Problems will be distributed among participants before the course.

The second lecture will last 90 minutes. The following topics will be discussed during the lecture.

(1) The Smith Theory for group actions on contractible CW complexes.

(2) Construction of contractible CW complexes with group actions.

(3) Equivariant K-Theory and extension of vector bundles with group actions.

(4) Thickening of CW complexes to smooth manifolds in the equivariant setting.

(5) The converse to the Smith Theory for group actions on contractible manifolds.

The Problem Session will be run with the help of **Wojciech Politarczyk**

Homotopy Theory of Lie groups and their Classifying Spaces

Stefan Jackowski

Module 1 – Lecture 1

(1) Lie groups, homomorphisms and linear representations. Irreducible representations.‘

(2) Maximal tori in compact Lie groups.

(3) Characters of representations. Ring of virtual characters. The Weyl theorem.

(4) Actions of Lie groups. Homogeneous spaces (orbits) and equivariant maps.

(5) Classifying spaces of topological groups and maps induced by homomorphisms.

(6) Homotopy classification of maps between classifying spaces of discrete groups.

(7) The Dwyer–Zabrodsky–Nottbohm theorem on homotopy classification of maps from the classifying spaces of p–toral groups to the classifying space of compact Lie groups.

(8) Linear representations vs. homotopy representations of compact Lie groups.

Module 2 – Problem Session

Participants will be split into groups of no more than 15 people, according to their familiarity with the subject. Each group will be working on problems related to Lecture 1 and preparation to Lecture 2. Problems will be announced before the course begins.

Module 3 – Lecture 2

(1) Localization and completion in homotopy theory. The Sullivan arithmetic square.

(2) Classifying spaces of small topological categories. Homotopy colimits.

(3) Decompositions of the classifying spaces of compact Lie groups into homotopy cilimit of the classifying spaces of its subgroups.

(4) Obstruction theory.

(5) Exotic maps between classifying spaces. Unstable Adam operations.

(6) Semi-ring of homotopy representations and its Grothendieck ring.

(7) Very exotic maps and open question on homotopy representation theory of compact Lie groups.

The Problem Session will be run with the help of **Wojciech Lubawski**