On the Spin and Spin$^\mathrm{C}$ Structures

Michał Banacki

The main goal of the lecture is to give an elementary introduction to the theory of spin and spin$^\mathrm{C}$ structures. Evoked subject has been broadly studied not only from the purely mathematical perspective, but also in the context of theoretical physics.

Starting from the concept of $Spin(n)$ and $Spin^C(n)$ groups we define the notions of spin and spin$^\mathrm{C}$ structures on $SO(n)$-principal bundle and give some examples. Then we recall the idea of second Stiefel–Whitney class and use it to describe conditions for existence of discussed structures.

Algebraic topology applied to evasiveness of graph properties

Maciej Gawron

Many graph properties (e.g., connectedness, containing a complete subgraph) are known to be difficult to check. The evasiveness conjecture states that any non-trivial monotone property (i.e. a property closed under removing edges) P of graphs on a fixed set of n vertices is evasive, which means that given an unknown graph G on the n vertices and allowed to ask whether a given edge belongs to G, we need in the worst case to ask about all possible $n(n-1)/2$ edges in order to determine whether G has the property P or not. This conjecture is unproven, but a lot of progress has been made. Starting with the work of Kahn, Saks, and Sturtevant in 1984, topological methods have been applied to prove partial results on the Karp conjecture. In this talk we will present central proofs from the paper of Kahn, Saks and Sturtevant and survey some more recent results on evasivness.

Let V be a set of size n, and let $G(V)$ denote the set of undirected graphs on V. A graph property is a function

$$f : G(V) \to \{0, 1\}$$

which is such that whenever two graphs G_1, G_2 are isomorphic then $f(G_1) = f(G_2)$. A graph G "has property f" if $f(G) = 1$. A graph property is called monotone it its always preserved by the addition of edges. To every monotone graph property f we associate some abstract simplicial complex Γ_f in such a way that if f is nonevasive then Γ_f has a certain topological property called collapsibility. The condition of collapsibility for a simplical complex Δ implies that the reduced homology groups of Δ are trivial. This allows us to use topological fixed-point theorems. Using the Olivers Fixed Point theorem we are able to prove The evassivness conjecture when n is prime power.

Depicting a Codimension-Two Smooth Embeddings of Surfaces

Michał Jabłonowski

Surfaces that are smoothly embedded in the four-dimensional euclidean space can be depicted in several ways. We will describe techniques of investigation, by examining multiple or critical points produced by mapping of a given embedding into a lower codimensional ambient space. The most popular corresponding descriptions are by a broken surface diagram, by a motion picture, by a marked vertex diagram and a resulting banded link presentation. We will present moves that relate ambiently isotopic surfaces with different diagrams. We will give examples of given constructions for main classes of knotted surfaces, to which the classification problem is still open.

Entropy and dynamics of hyperbolic measures

Martha Łącka

We will describe some of the consequences of the existance of hyperbolic measures for $\mathcal{C}^{1+\alpha}$ diffeomorphisms of compact surfaces. The talk will be based mainly on the works by Katok and Mendoza.

Some Remarks on Quandles and Their Applications

Anna Parlak

In 1982 David Joyce introduced the notion of a quandle, that is an algebraic structure satisfying certain conditions corresponding to Reidemeister moves. This knot-inspired object has utility not only in defining knot invariants but also outside knot theory (e.g. Dehn quandle of a surface). We begin with the definition of a quandle and show how it appears naturally in the study of knots. Then we explain the Wirtinger presentation of the knot group (the fundamental group of a knot complement in $\mathbb{R}^3$) and demonstrate how it is associated with the knot quandle. The lecture is introductory in scope and assumes listeners has no or little familiarity with knot theory.

A new concept of stability

Jakub Pawlikowski

Stability occurs very often in topology - one can think about the beginning of this notion (Freudenthal theorem), about stable homotopy groups of spheres, about Bott periodicity... On last International Congress of Mathematicians Benson Farb has presented his discovery - it was about structure of configuration spaces. They aren't stable in normal, old sense (they have larger and larger cohomology groups when the dimension increases), but these groups are stable when they are considered as representations of symmetric group (which, of course, acts on the configuration space). I'll present the historical context of stability problems, tell something about representations of symmetric groups, define configuration spaces and introduce this fantastic new idea of stability in topology.

The J-homomorphism and its applications

Igor Sikora

The J-homomorphism is a certain homomorphism

$$J : \pi_n(O(n)) \to \pi_{n+i}(\mathbb{S}^n),$$

which can be useful in computing homotopy groups of spheres. My aim during this lecture is to define this homomorphism and show its basic applications.

Morse theory

Marcin Sroka

Morse theory enables one to study the topology of a manifold by considering differentiable functions on that manifold. We will give a systematic introduction to the theory and prove some important theorems such as Reeb's theorem. It states that if there exists a Morse function on compact manifold with only two critical points than the manifold is homeomorphic to the sphere. We will give an application for it during the process of constructing an exotic sphere.