ABSTRACTS OF LECTURES

Edge-Decompositions of Graphs

János Barát

A graph G consists of a finite set $V(G)$ of vertices and $E(G)$ of edges, where the edge set is a subset of pairs in $V(G)$. Let $\mathcal{H}$ be a collection of graphs. A graph G has a $\mathcal{H}$-decomposition if the edges of G can be divided into subgraphs each of which is isomorphic to a graph in $\mathcal{H}$. Often $\mathcal{H}$ is a single graph H or all graphs in $\mathcal{H}$ have the same number of edges. The H-decompositions are widely studied when G is a complete graph. If G is not complete, then it may be hard to find H-decompositions. Indeed, if H has at least three edges, then the problem of deciding if a graph G has an H-decomposition is NP-complete.

It is natural to ask what edge-connectivity may guarantee certain decompositions. We would like to survey the results in this context from the last 50 years or so. The development of the subject is somewhat highlighted by the following

Conjecture. (Barát and Thomassen 2006)

For each tree T, there exists a natural number k_T such that the following holds: If G is a k_T-edge-connected graph such that $|E(T)|$ divides $|E(G)|$, then the edges of G can be divided into parts, each of which is isomorphic to T.

We will pose numerous open problems, many of which should be suitable for a student's attack.

Harmonious Coloring of Hypergraphs

Bartłomiej Bosek

A *harmonious* coloring of a hypergraph is a coloring of its vertices such that

- two different vertices contained in common hyperedge have different colors,
- sets of colors of all vertices from two different hyperedges are different sets.

We prove that every k-uniform hypergraph, with maximal degree Δ and with m hyperedges, is harmoniously colored by

$$O\left(\frac{k}{k-1}\sqrt[k]{(k-1)k!\Delta m}+\Delta^2+(k-1)\Delta\right)$$

colors. This is almost tight, since the obvious lower bound is of order $\Omega(\sqrt[k]{k!m})$. The proof uses *entropy compression* argument – a novel method inspired by the algorithmic version of the Lovász Local Lemma due to Moser and Tardos. We will also discuss several related problems, for instance *legitimate* coloring of hypergraphs, *strong* coloring of graphs, etc.

Specification-like Properties in the Symbolic Dynamics

Martha Łącka

The specification property was introduced by Bowen to derive ergodic properties of dynamical systems. Later this notion has been generalized in many different ways. In the speech I will show how to translate the definitions of specification-like properties into the language of symbolic dynamics. I will also mention some already- known spectacular results about systems with specification-like properties.

Permanents and the Van der Waerden Conjecture

Dariusz Matlak

We define permanent of a $n \times n$ matrix $A = [a_{ij}]$, by:

$$\mathrm{per}A := \sum_{\pi \in S_n} a_{1\pi(1)} \cdot a_{2\pi(2)} \cdot \ldots \cdot a_{n\pi(n)}$$

This simple function gives a valuable stuff useful while proving combinatorial identities, and computing number of matchings in a graph. There are elementary theorems concerned with relationships between row sums of a matrix and the permanent of that matrix. One conjectured by Minc in 1967 yields to an upper bound in terms of row sums. The second, known as Van der Waerden conjecture gives lower bound for doubly stochastic matrices.

I will describe both conjectures (now theorems) with sketches of proofs and interesting consequences.

Combinatorial Limits

Daniel Kráľ

The theory of combinatorial limits has attracted a substantial interest among combinatorialists and it led to opening new exciting links to other areas of mathematics and computer science. Inside combinatorics, the flag algebra method, which is closely linked to combinatorial limits, led to progress on many hard open problems. In this talk, we will survey these exciting recent developments. We will start with presenting basic concepts from the theory of limits of dense structures (such as graphs with quadratically many edges) and limits of sparse structures (such as graphs with bounded degrees), explore the connection of the dense case to the flag algebra method and relate different notions of convergence in the sparse case.

Ramsey Theory on the Integers

Jarosław Grytczuk

In 1927 van der Waerden proved that in any finite coloring of natural numbers one may find arbitrarily long arithmetic progressions with all terms of the same color. This result inspired lots of research culminating in such spectacular achievements like Szemeredi's theorem, or the theorem of Green and Tao. It also influenced a significant development of other areas of mathematics, especially Topological Dynamics and Ergodic Theory, leading to such great discoveries like the celebrated Furstenberg's Correspondence Principle.

I will present some my favorite problems in this area. One of the newest is Bosek-Pálvölgyi Conjecture stating that for every $k \in \mathbb{N}$ there is a k-coloring of natural numbers such that for every $a \in \mathbb{N}$ the set $\{a, 2a, \ldots, ka\}$ is rainbow. It is not hard to prove that conjecture is true for $k = p - 1$, where p is a prime number. It has also been confirmed for all $k \leq 194$. The problem has intriguing connections to Latin squares, tilings of $\mathbb{Z}^n$, and the famous Graham's greatest common divisor problem.

On the Intersection Graph of Modules

Marta Nowakowska

The intersection graph was defined and studied for many algebraic structures. In 1964, Bosak introduced this graph for semigroups and Csákány and Pollák in 1969, studied it for finite groups.

In this talk we focus on the intersection graph of modules and rings. For a given module M, the intersection graph $G(M)$ of M is defined as the simple, undirected graph whose set of vertices consists of non-trivial submodules of M and two distinct submodules N_1, N_2 are adjacent if and only if $N_1 \cap N_2 \neq 0$. For a given associative ring R the intersection graph of R is defined as $G({}_RR)$, where ${}_RR$ denotes the left R-module R. The first remarkable results in this area was obtained by D.Bertholf and G.Walls in *Graphs of finite abelian groups.* From their results it, in particular, follows that if A, B are finite abelian p-groups for a prime p, then $A \simeq B$ if and only if $G(A) \simeq G(B)$. This result shows that using graph-theoretic concepts one can get quite important information on modules.

The aim of the talk is to survey some results on the relationship between properties of a module M and the graph $G(M)$ obtained jointly with E. R. Puczyłowski. Our studies were motivated by the results obtained in S. Akbari, R. Ninakdish and M. J. Nikmehr: *Some results on the intersection graph of rings.* and S. Akbari, H. A. Tavallaee and S. Khalashi Ghezelahmad: *Intersection graph of submodules of a module.* and the problem raised in the first of them asking for a description of rings whose intersection graph is infinite and which contain maximal left ideals of finite degree. We solved this problem and we also characterized modules and rings with some other properties of their intersection graph. These results concern in particular the clique and chromatic numbers of $G(M)$. For instance, we described modules whose intersection graph is infinite and has finite clique number and proved that for such modules the clique and chromatic numbers coincide.

The Polynomial Method in Combinatorics

Marcin Lara

The polynomial method is a technique in combinatorics and number theory for controlling a relevant set of points by comparing it with the zero set of a suitably chosen polynomial, and then using tools from algebraic geometry (e.g. Bezout's theorem) on that zero set. The polynomial method has been used many times in different fields of mathematics; in combinatorics, the nullstellensatz of Alon is relatively early use of it. More recently, it underlies proof of the Kakeya conjecture over finite fields and near-complete solution to the Erdős distance problem in the plane, and can be used to give a short proof of the Szemerédi-Trotter theorem. I will present the method and some of the above applications.

Gra w marynarza – Counting-out Game with Modular Arithmetic

Piotr Karwala

Lecture is about a simple counting-out game, called in Polish "gra w marynarza" and issue of equal chances to win the game. The problem involves remainder of sum of integer (valued) random variables. Main result of the lecture provides sufficient condition for discrete uniform distribution of such defined random variable. Lecture contains:

- Explanation of rules of the game,
- Model of the game with classical probability definition (sufficient and necessary conditions for equal chances to win),
- General model with integer random variables (sufficient condition, general theorem).

Main theorem: (Let $r_m(x)$ denote remainder: $x = qm + r_m(x)$) Let $n, m \in \mathbb{N}_*$. Suppose $\xi_1, \xi_2, \ldots, \xi_n$ are integer random variables and there exists $i_0 \in \{1, 2, \ldots n\}$ such that

(1) $r_m(\xi_{i_0})$ has discrete uniform distribution,
(2) random variables $r_m(\xi_{i_0})$ and $r_m(\sum_{i \neq i_0} \xi_i)$ are independent.

Then $r_m(\sum_{i=1}^{n} \xi_i)$ has discrete uniform distribution.

The Motorway Problem from the Physicists Point of View

Dominik Wrana

In this short talk the motorway problem will be introduced. It is a marvellous application in optimisation and shortest-path problems. Along with the main concept of this subject, a couple of examples and solutions will be shown. The second part of the presentation will regard, surprisingly enough, the physics of soap bubbles. The main goal is to show an interconnection between mathematical abstract and a physical phenomenon. Most presumably it would be the only lecture making use of the actual real life demonstrations.

Equilateral Sets in Finite Dimensional Normed Spaces

Tomasz Kobos

Let X be a real n-dimensional vector space equipped with a norm $||\cdot||$. We say that a set $S \in X$ is *equilateral*, if there is a $p > 0$ such that $||x - y|| = p$ for all $x, y \in S, x \neq y$. By $e(X)$ we shall denote the *equilateral dimension* of the space X, defined as the maximal cardinality of a equilateral set in X. Equivalently, an equilateral set in a normed space X corresponds to a system of a pairwise touching translations of the unit ball of X. Consequently, equilateral dimension is equal to the maximal possible number of a pairwise touching translations of the unit ball. In this manner problems concerning equilateral sets can be classified as the problems in the field of discrete geometry.

The main goal of the talk is to present basic results concerning equilateral sets in finite dimensional normed spaces. It is not difficult to show that the equilateral dimension of n-dimensional space equipped with the Euclidean norm is equal to $n + 1$, and it is equal to 2^n for the ℓ_∞ norm. It is known that 2^n is, in fact, an upper bound for the equilateral dimension of any normed space X of dimension n. However, the equilateral dimension of the other ℓ_p^n spaces is not known in most cases.

It is also widely conjectured that the equilateral dimension of any n-dimensional normed space has an equilateral set of at least $n + 1$ points. We shall discuss results concerning this conjecture, also in the setting of concrete classes of spaces.

Sequences Generated by Finite Automata

Jean-Paul Allouche

It is not easy to define what a "random" sequence is. The opposite point of view could be to introduce "non-random" sequences and in particular "algorithmic" sequences. We will present a class of "easy" algorithms to generate sequences, namely finite automata. They generate the so-called automatic sequences. We will show that automatic sequences can be found in many fields of mathematics. In particular we will try to explain their interest in number theory, harmonic analysis, combinatorics... We will also allude to the use of these sequences in physics, computer science, or even music.

Hankel Determinants of the Cantor Sequence

Piotr Miska

The Cantor sequence $\{c_n\}_{n\in\mathbb{N}}$ is defined by the formula:

$$c_n = \begin{cases} 0 \text{ when '1' occurs in 3-ary expansion of n,} \\ 1 \text{ otherwise.} \end{cases}$$

This sequence is an automatic sequence, i.e. it can be generated by a finite automaton. It is associated with the Cantor ternary set.

For a sequence of complex numbers $\{u_n\}_{n\in\mathbb{N}}$ we define (p,n)-order Hankel matrix by the formula $H_n^p = (u_{p+i+j-2})_{1\le i,j\le n}$. The determinant $|H_n^p|$ of this matrix is called the Hankel determinant.

Allouche, Peyrière, Wen and Wen studied the properties of Hankel determinants $|\mathcal{E}_n^p|$ of the Thue-Morse sequence and they proved that the two-dimensional sequence $\{|\mathcal{E}_n^p| \pmod 2\}_{p,n\in\mathbb{N}}$ is 2-automatic.

During this talk we will present results of Wen and Wu concerning Hankel determinants $|\Gamma_n^p|$ of the Cantor sequence. We will show that for each positive integer p the sequence $\{|\Gamma_n^p|$

(mod 3)$\}_{n\in\mathbb{N}}$ is periodic and the two-dimensional sequence $\{|\Gamma_n^p|$ (mod 3)$\}_{p,n\in\mathbb{N}}$ is 3-automatic. We use this results to prove the existence of Padé approximants for the generating function of the Cantor sequence and to show that irrationality exponent of the Cantor number, i.e. the number with Cantor sequence as its b-ary expansion, equals 2.

A Characterization of the Ramsey Property of Families of Finite Subsets of the Positive Integers

Michał Świętek

Recall that the classical Ramsey theorem asserts that for every positive integer k and every partition $\mathcal{F}_1 \bigcup \mathcal{F}_2 = \mathbb{N}^{[k]}$ of the set of k-elements subsets of positive integers there exist an infinite set $M \subset \mathbb{N}$ and $i \in \{1, 2\}$ such that $M^{[k]} \subset \mathcal{F}_i$. A family of finite subsets of $\mathbb{N}$ is called *Ramsey* if it satisfies the thesis of Ramsey theorem.

During the talk I would like to present the following Nash-William's characterization of the Ramsey property: a family $\mathcal{F}$ of finite subsets of $\mathbb{N}$ is Ramsey iff there exists an infinite set $M \subset \mathbb{N}$ such that the family $\mathcal{F} \cap \mathcal{P}(M)$ is Sperner (a family of finite subsets of $\mathbb{N}$ is called *Sperner* if $s \not\subseteq t$ for every pair $s \neq t \in \mathcal{F}$).

This theorem is a starting point of carrying Ramsey theory to infinite dimension, which in turn has a number of applications in other mathematical branches, i.e. Geometry of Banach spaces or Theory of well-quasi-orderings. Another interesting fact about the above characterization is that its proof introduces the first instance of so called *combinatorial forcing*.

Thue-Morse Sequence, Identities and Partition Functions

Maciej Ulas

In the first part of the lecture we recall main properties of the Thue-Morse sequence $\mathbf{t} = \{t_n\}_{n\in\mathbb{N}}$ and its (ordinary) generating function $F(x) = \sum_{n=0}^{\infty} t_n x^n$. We have $t_n = (-1)^{s_2(n)}$, where as usual, $s_2(n)$ is the sum of digits of the (unique) binary expansion of the integer n. In the second part of the lecture we are interested in some generalizations of the identity of Nieto

$$\sum_{i=0}^{2^n-1} t_i(x+i)^n = (-1)^n n! 2^{\frac{n(n-1)}{2}}$$

and Bateman and Bradley

$$\sum_{i=0}^{2^n-1} t_i(x+i)^{n+1} = (-1)^n (n+1)! 2^{\frac{n(n-1)}{2}} (x + \frac{2_n - 1}{2}).$$

More precisely, we consider the sequence of polynomials

$$F_{m,n}^{\mathbf{u}}(x) = \sum_{i=0}^{k^n-1} \zeta_k^{s_k(i)} (x + \mathbf{u}(i))^m,$$

where

$$\mathbf{u}(ki+j) = P(\mathbf{u}(i)) + jq \text{ for } j = 0, 1, ..., k-1,$$

and $q \in V$, Here, V is a finitely dimensional vector space over the field K and $P : V \to V$ is a linear endomorphism. Moreover, $s_k(n)$ is the sum of digits of the expansion of the integer n in base k. We reduce the computation of $F_{m,n}^{\mathbf{u}}(x)$ to the computation of $F_{m,n}^{\mathbf{u}}(0)$. Next, we give an explicite expression of $F_{m,n}^{\mathbf{u}}(0)$. We also prove some related results which are of independent interests.

Finally, in the last part of the lecture we present some result concerning the arithmetic properties of coefficients of power series expansions of the function $F(x)^t = \sum_{n=0}^{\infty} f(t, n)x^n$. In particular we present characterization of the solutions of the equations $f(2, n) = 0$ and $f(3, n) = 0$. Next, we observe that if t is negative integer then $f_{t,n}$ counts the number of binary partitions of n

with $|t|$ colors. In particular, we prove that if $t = 1 - 2^m$, then $1 \leq v_2(f(t,n)) \leq 2$ for $n \geq m$ and get expression for $v_2(f(t,n))$, where as usual $\ni_2$ (n) is 2-adic valuation of n. We also present some congruence relations for the coefficients $f(t,n)$ in this case.

Twins in Words and Permutations

Maciej Gawron

For a word w let $f(w)$ be the largest integer k such that there are two identical, disjoint subsequences (called twins) of length k in w. We will investigate asymptotic of the following function

$$T(n, \Sigma) = \min\{f(w) | w \in \Sigma^n\},$$

where Σ is a finite alphabet. As a main theorem we will prove that $2T(n, \{0,1\}) = n - o(n)$. The proof is based on regularity lemma for words.

In the sequel we will consider analogous problem for permutations. Let $\sigma \in S_n$ be a permutation over the alphabet $\{1, 2, \ldots, n\}$. Two disjoint subsequences $\{a_i\}_{i=1}^k$, $\{b_i\}_{i=1}^k$ of $(1, 2, \ldots, n)$ are called twins if the following condition

$$\sigma(a_i) \leq \sigma(a_j) \iff \sigma(b_i) \leq \sigma(b_j)$$

holds for all $i, j = 1, 2, \ldots, k$. Let $g(\sigma)$ be the largest integer k such that there are twins of length k in σ. We will give some bounds on the asymptotic of the following function

$$P(n) = \min\{g(\sigma) | \sigma \in S_n\}.$$

In order to prove our bounds we will use probabilistic methods, Lovász local lemma, and Erdős-Szekeres theorem.

On Aztec Diamonds Tilings

Igor Sikora

I want to say a few words about tilings of Aztec Diamonds – especially about a number of such tilings in relation to size of an Aztec Diamond and about random tilings in large Aztec Diamonds (it turns out that "expected shape" of such tilings in the interior of a diamond is a circle).

The Minimum Cut Problem and Connectivity of Generalized Random Graphs

Justyna Tabor

‘ The minimum cut in a graph is a set of edges of minimum sum of weights, whose deletion disconnects the graph. There are few efficient algorithms finding the minimum cut in a given weighted graph. I will dedicate the first part of my lecture to show a random algorithm of Karger and Stein. In the second part I will demonstrate a generalized random graph model and I will finish giving the sufficient condition for connectivity of this graph using the result of Karger and Stein.